RDFS with Attribute Equations via SPARQL Rewriting

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Place de la Comédie, Montpellier

avg. temp. in May in °C

population

area

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Place de la Comédie, Montpellier

avg. temp. in May in °C

population

area

CO₂ emissions per person

avg. temp. in May in F

population density

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Use equations to infer missing numbers What is the population density of Montpellier?

Montpellier

population: 252 998 area: 56 880 000 m² population density: ??? in people/km²



- Can we infer population density from given data? computations not supported by Semantic Web reasoners
- How can we get area in km²? unit conversion by computation

RDFS with Attribute Equations via SPARQL Rewriting the big picture



What is the population density of Montpellier? written in SPARQL

SELECT ?dens

WHERE { :Montpellier :populationDensity ?dens .}

```
SELECT ?city ?dens
WHERE {
    :Montpellier :populationDensity ?mdens .
    ?city rdf:type :City ;
        :populationDensity ?dens .
    FILTER(?dens > ?mdens)
}
```

RDFS with Attribute Equations via SPARQL Rewriting the big picture



We need RDFS for integrating different sources



- Unify RDFS properties of different data sources
- Use unified name for population dbp:populationTotal rdfs:subpropertyOf :population geo:population rdfs:subpropertyOf :population
- Use already implemented RDFS reasoners which allow SPARQL queries

RDFS with Attribute Equations via SPARQL Rewriting the big picture



Syntax RDFS and equations converting between RDFS and DL_{RDFS}^E

$A_1 \sqsubseteq A_2$	A_1 rdfs:subClassOf A_2
$\exists P \sqsubseteq A$	P rdfs:domain A
$\exists P^- \sqsubseteq A$	P rdfs:range A
$\exists U \sqsubseteq A$	U rdfs:domain A
$P_1 \sqsubseteq P_2$	P_1 rdfs:subPropertyOf P_2
$U_1 \sqsubseteq U_2$	$U_1 rdfs:subPropertyOf U_2$
$U_0 = f(U_1, \dots, U_n)$	U_0 definedByEquation "f $(U_1, \dots, U_n)''$
A(x)	$x \operatorname{rdf:type} A$

R(x,y)	x R y
U(x,q)	$x \ U \ "q" \ owl$: rational

Syntax RDFS and equations converting between RDFS and DL _{RDFS} E		
dbp:population	rdfs:domain dbp:populatedPlace.	
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U(x,q)	$x \cup "q" \sim owl: rational$	
	:Montpellier dbp:population 252998 .	

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$egin{array}{l} A(x)\ R(x,y) \end{array}$	x dbp:populationDensity :definedByEquation "dbp:population / dbp:area".	
U(x,q)	$x \cup "q" \frown owl:rational$	
	:Montpellier dbp:population 252998 .	

- RDFS + attributes: usual DL model theoretic semantics
- For an equation, infer a new value if all other attributes of the equation are given and there is no division by zero then the computation result is the new attribute value

•
$$U_0 = f(U_1, \ldots, U_n)$$
 satisfied in \mathcal{I}

if
$$\forall x, y_1, \dots, y_n (\bigwedge_{i=1}^n (x, y_i) \in U_i^{\mathcal{I}}) \land \text{defined}(f(U_1/y_1, \dots, U_n/y_n))$$

 $\Rightarrow (x, \text{eval}(f(U_1/y_1, \dots, U_n/y_n)) \in U_0^{\mathcal{I}})$

 Query answers are not necessarily finite ABoxes inconsistent with equations

dbp:populationDensity :definedByEquation "dbp:population / dbp:area".

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- For an equation, infer a new value if all other attributes of the equation are given and there is no division by zero then the computation result is the new attribute value

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Query answers are not necessarily finite
 ABoxes inconsistent with equations

dbp:populationDensity :definedByEquation "dbp:population / dbp:area".

- RDFS + attributes: usual / L model theoretic semantics
- For an equation, infer a new value if all other attributes of the equation there is no division by zero then the computation result is the n
 U₀ = f(U₁, ..., U_n) satisfied in I

$$\begin{array}{c} \text{if } \forall x, y_1, \dots, y_n(\bigwedge_{i=1}^n (x, y_i) \in U_i^{\mathcal{I}}) \land \text{defined}(f(U_1/y_1, \dots, U_n/y_n)) \\ \Rightarrow (x, \text{eval}(f(U_1/y_1, \dots, U_n/y_n)) \in U_0^{\mathcal{I}} \end{array} \end{array}$$

Query answers are not necessarily finite
 ABoxes inconsistent with equations

dbp:populationDensity :definedByEquation "dbp:population / dbp:area".

- RDFS + attributes: usual / model theoretic semantics
- For an equation, infer a new value if all other attributes of the equation there is no division by zero then the computation result is the n
 Montpellier dbp:population 252998 :Montpellier dbp:area 56.88 .

•
$$U_0 = f(U_1, \dots, U_n)$$
 satisfied in \mathcal{I}

if $\forall x, y_1, \dots, y_n (\bigwedge_{i=1}^n (x, y_i) \in U_i^{\mathcal{I}}) \land \text{defined}(f(U_1/y_1, \dots, U_n/y_n))$

$$\Rightarrow (x, \operatorname{eval}(f(U_1/y_1, \dots, U_n/y_n)) \in U_0^{\mathcal{I}}$$

 Query answers are not necessarily finite ABoxes inconsistent with equations

:Montpellier dbp:populationDensity 4447.93.

Formulate equations as rules n rules for equations in n variables

Equation given for population density

 $area_{km2} = \frac{population}{popDensity}$

Formulate equation as rule

 $area_{km2}(X, A) \Leftarrow popDensity(X, PD), population(X, P), A = P \div PD.$

More rules needed to cover all directions

 $popDensity(X, PD) \iff population(X, P), area_{km2}(X, A), PD = P \div A.$ $population(X, P) \iff area_{km2}(X, A), popDensity(X, PD), P = A \times PD.$

Forward chaining often does not terminate because of rounding errors

 $popDensity(X, PD) \iff population(X, P), area_{km2}(X, A), PD = P \div A.$ $area_{km2}(X, A) \iff population(X, P), popDensity(X, PD), A = P \div PD.$ $population(X, P) \iff area_{km2}(X, A), popDensity(X, PD), P = A \times PD.$

- DBpedia: population 252 998, area 56.88 km²
- Apply rule: population density 4447.925...293
- Apply rules: population 252 997.999...999 and area 56.880...003
- Rules engine computes population density again: 4447.925...275



Naive backward chaining does not terminate unfolding of recursive rules blows up arbitrarily

 $popDensity(X, PD) \iff population(X, P), area_{km2}(X, A), PD = P \div A.$

 $population(X, P) \iff area_{km2}(X, A), popDensity(X, PD), P = A \times PD.$

 $area_{km2}(X, A) \Leftarrow popDensity(X, PD), population(X, P), A = P \div PD.$

To compute the population density query for population density makes no sense



Naive backward chaining does not terminate unfolding of recursive rules blows up arbitrarily

$$popDensity(X, PD) \Leftarrow population(X, P), area_{km2}(X, A), PD = P \div A.$$

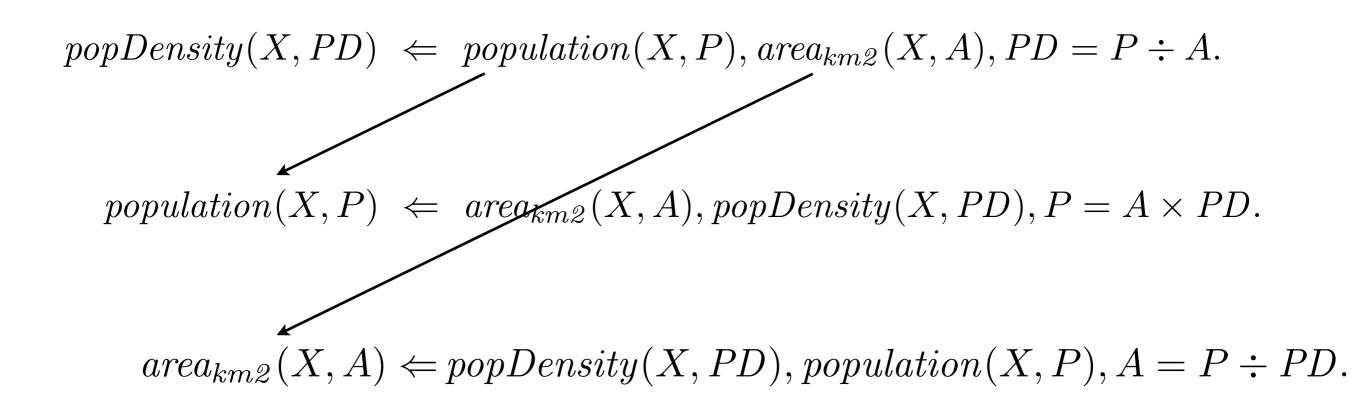
$$population(X, P) \Leftarrow area_{km2}(X, A), popDensity(X, PD), P = A \times PD.$$

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Naive backward chaining does not terminate unfolding of recursive rules blows up arbitrarily



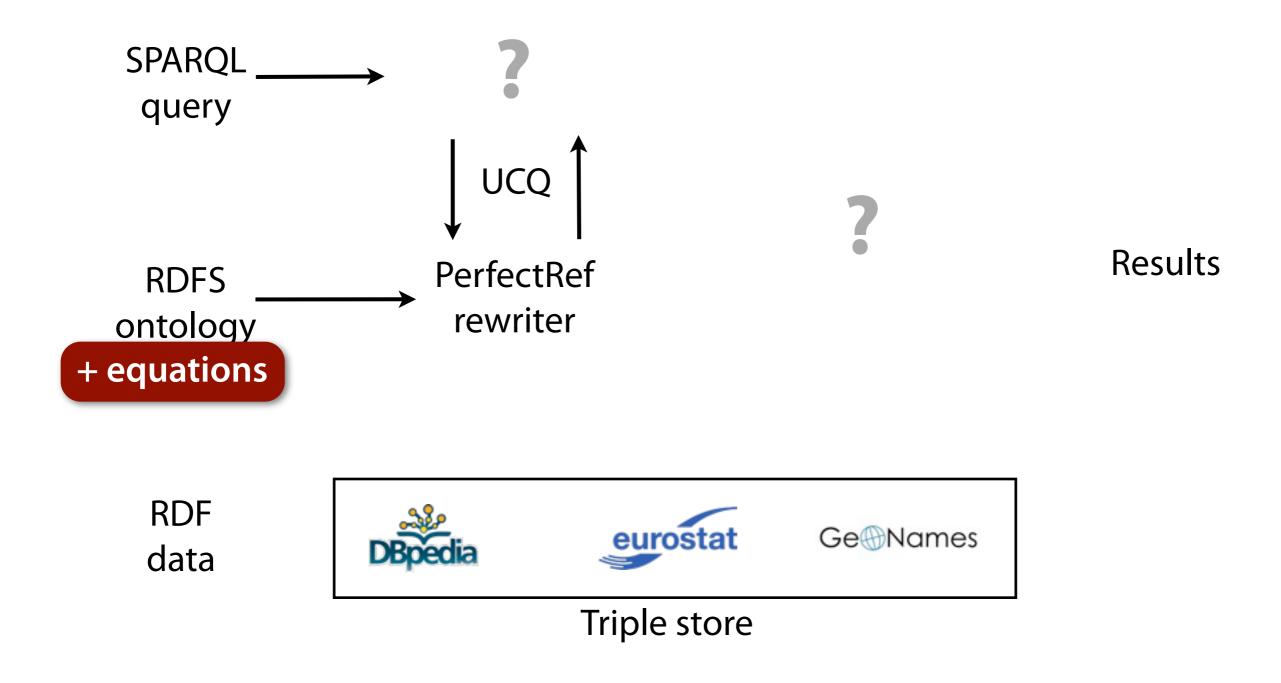
To compute the population density query for population density makes no sense



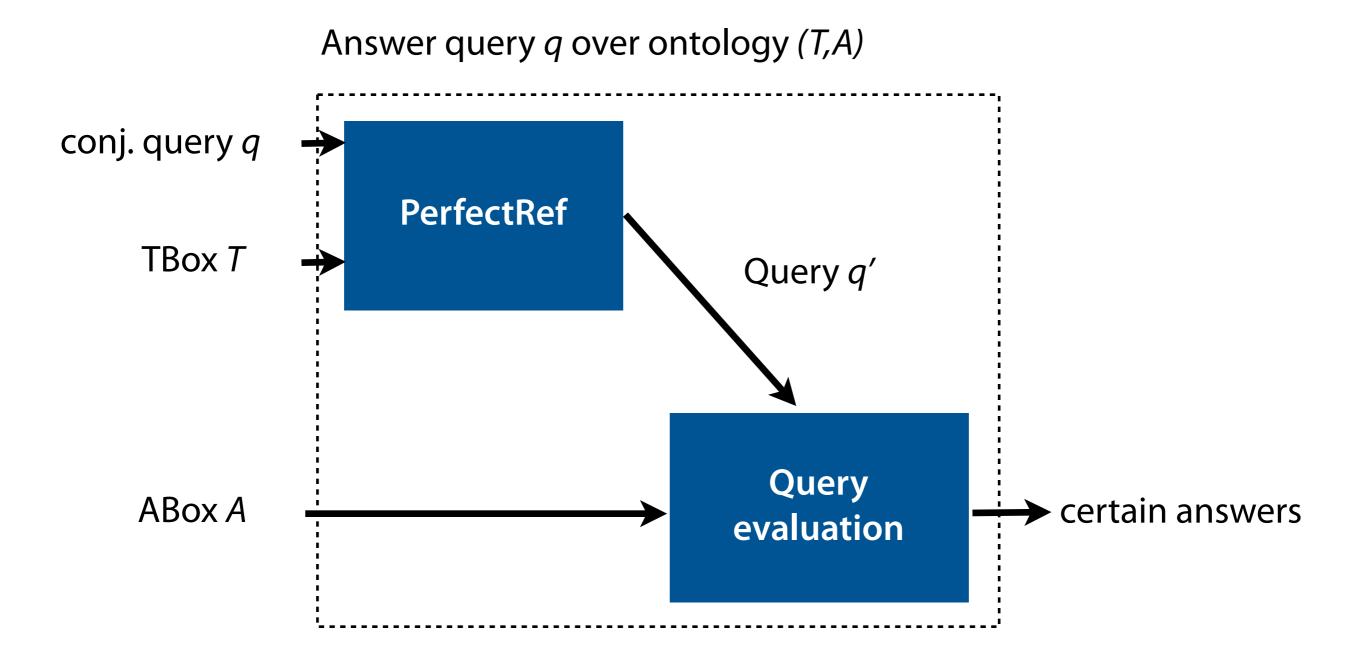
Rules are problematic for applying equations break the infinite series of rule applications

- Need to specify all directions of the equation not as intuitive and short as equations
- Forward chaining often does not terminate division or multiplication is often enough for non-termination implementation dependent
- Backward chaining does not terminate unfolding of recursive rules can blow up arbitrarily even for a single equation no termination
- We have to *break* the infinite series of rule applications

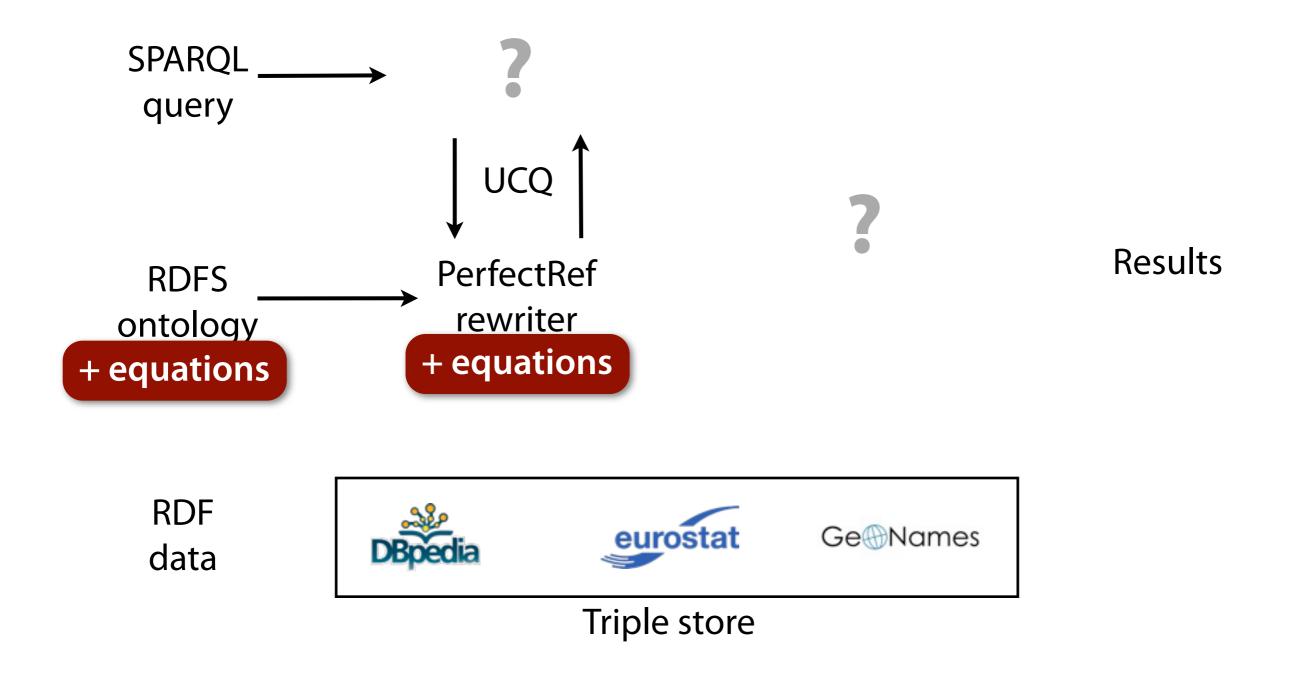
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Query answering in DL-Lite: PerfectRef Encode TBox in the query [Calvanese et al., 2009]

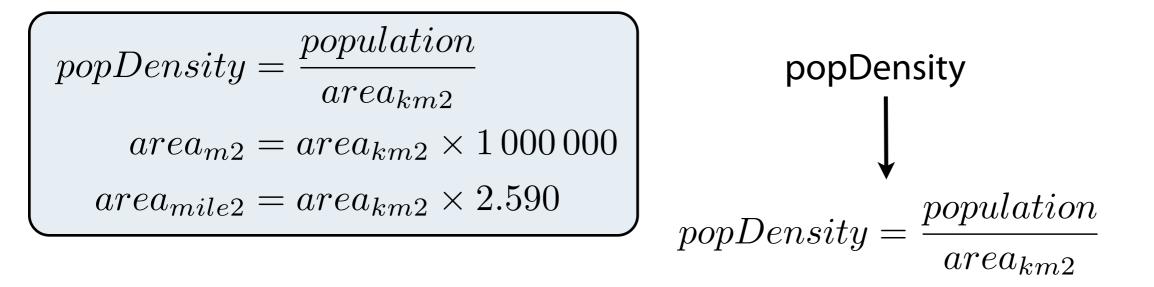


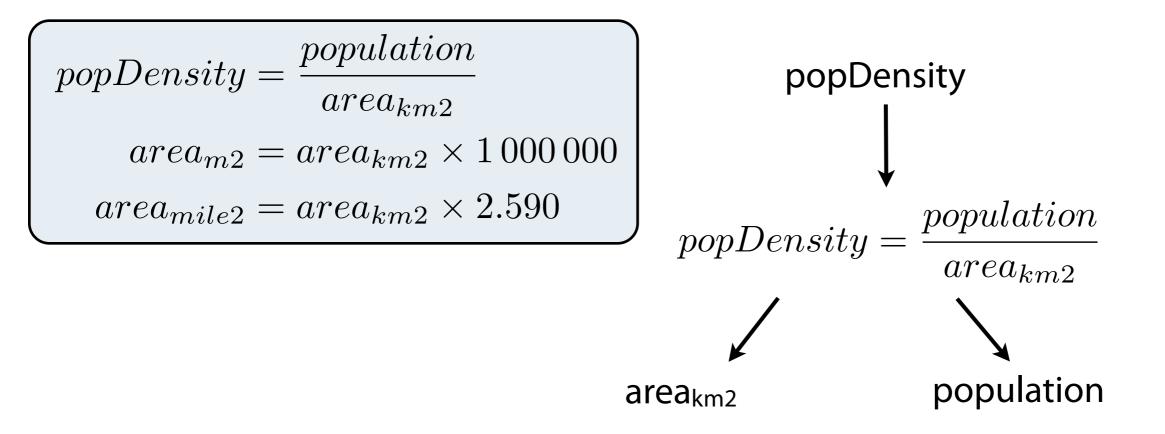
RDFS with Attribute Equations via SPARQL Rewriting the big picture

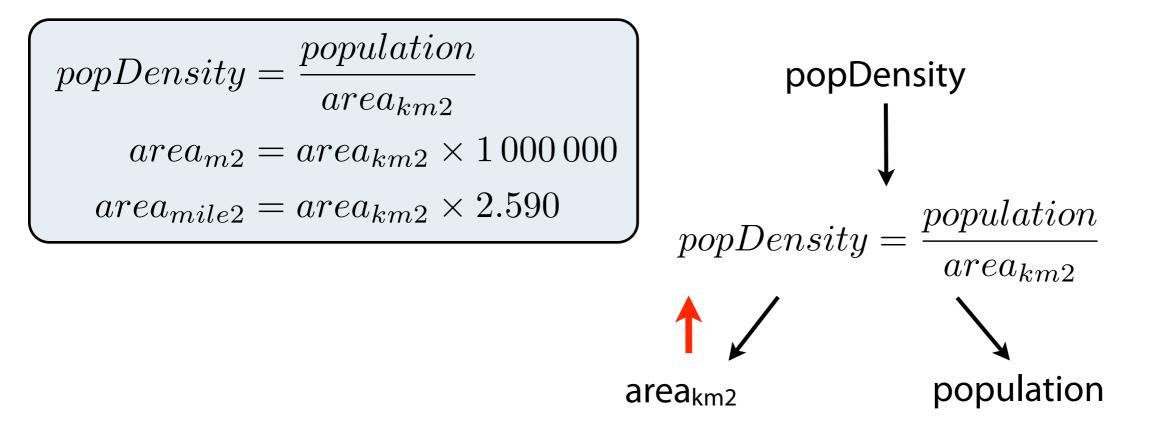


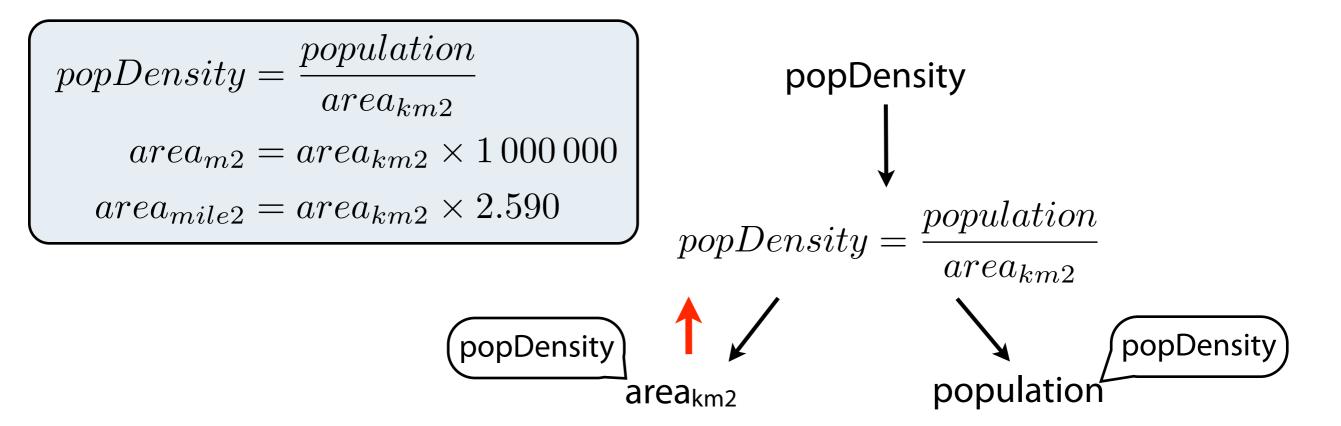
 $popDensity = \frac{population}{area_{km2}}$ $area_{m2} = area_{km2} \times 1\,000\,000$ $area_{mile2} = area_{km2} \times 2.590$

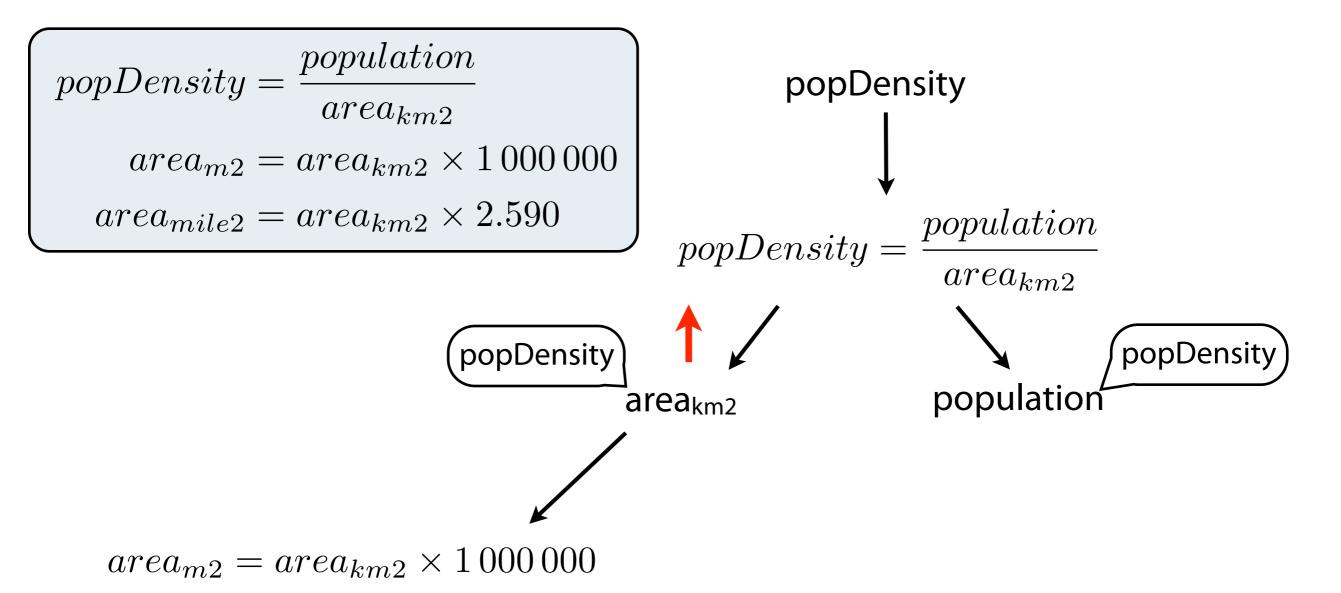
popDensity

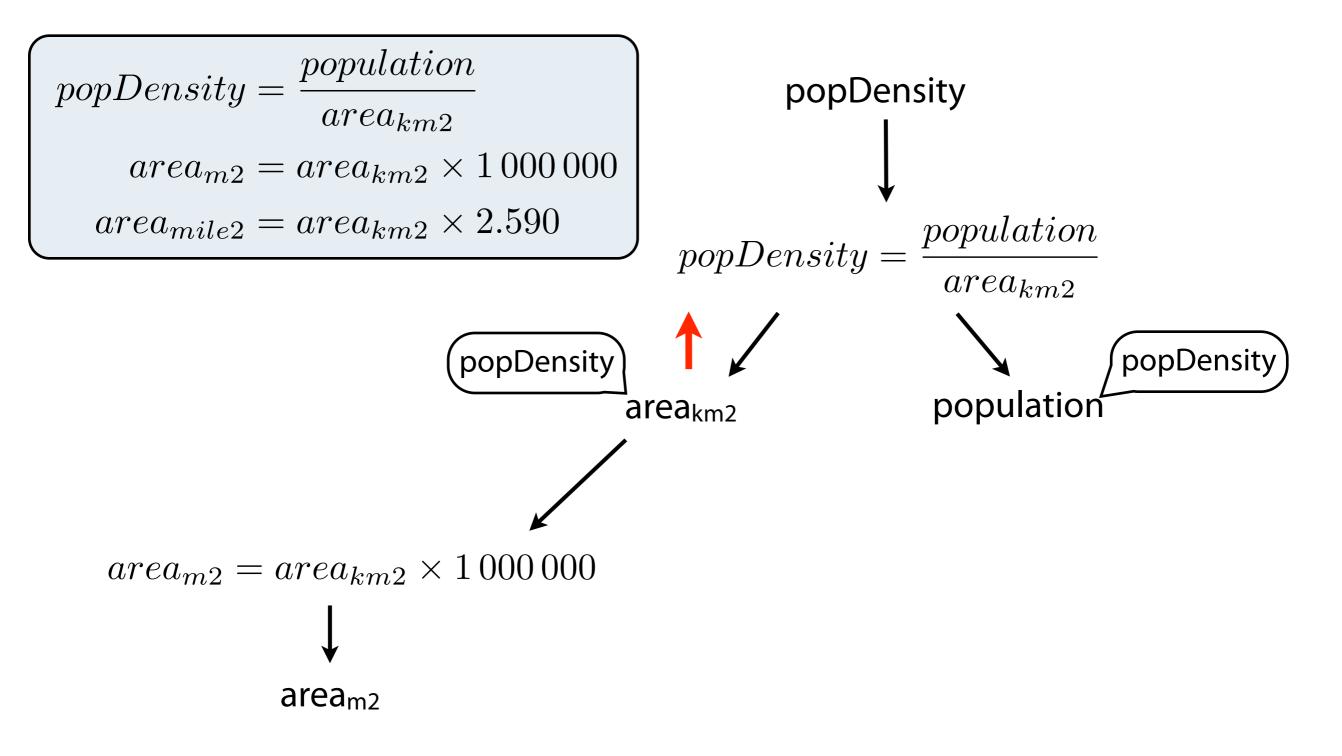


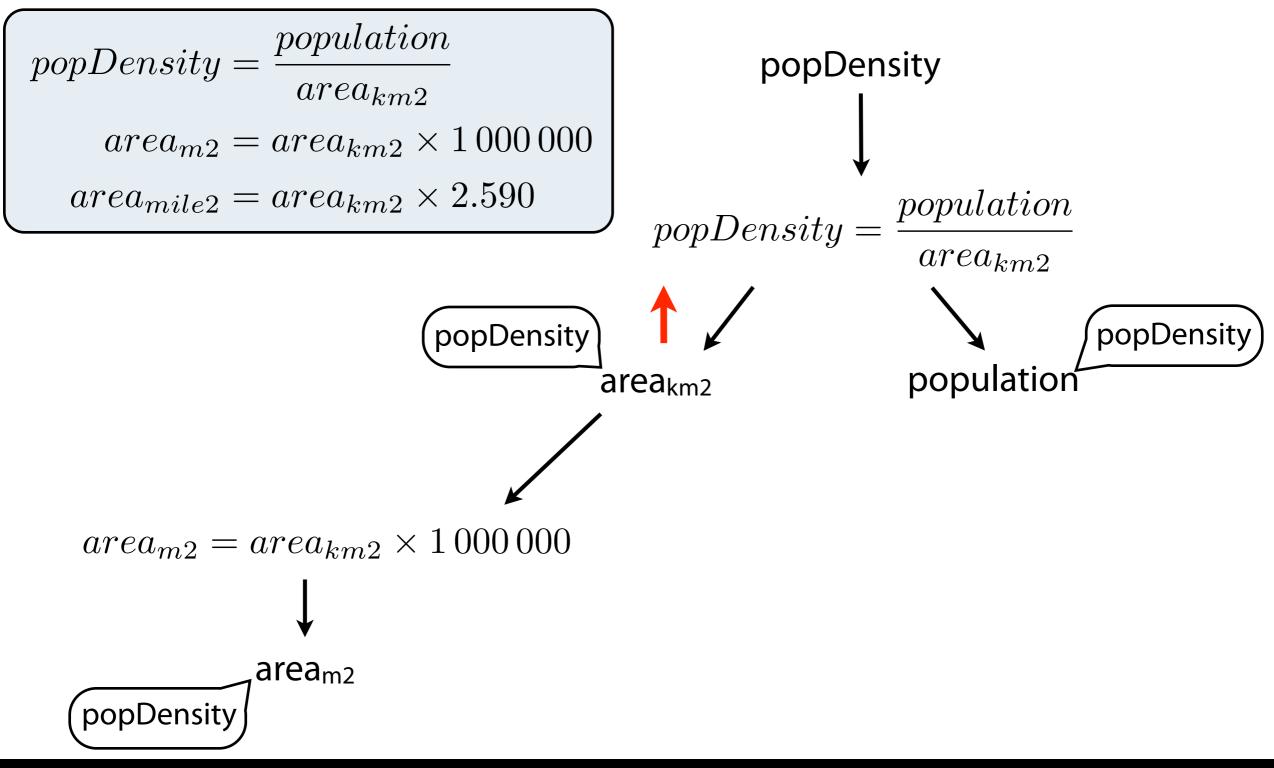


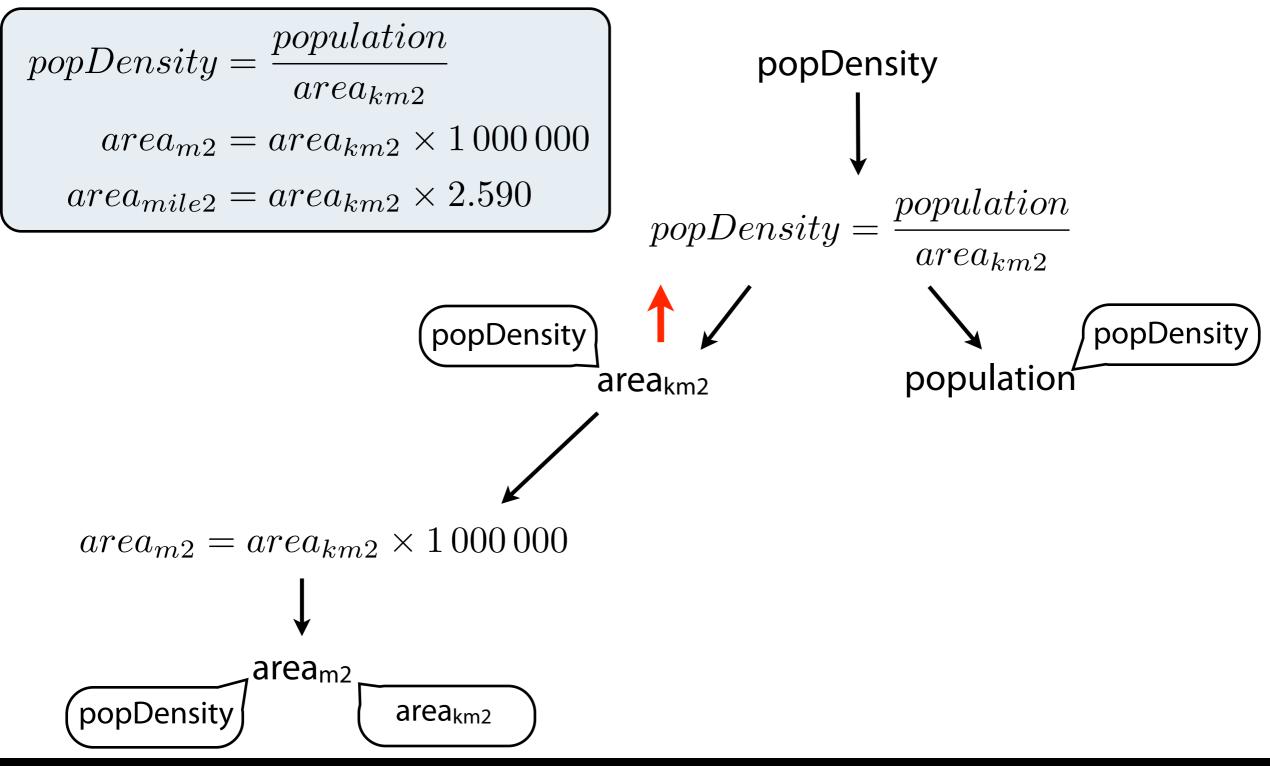


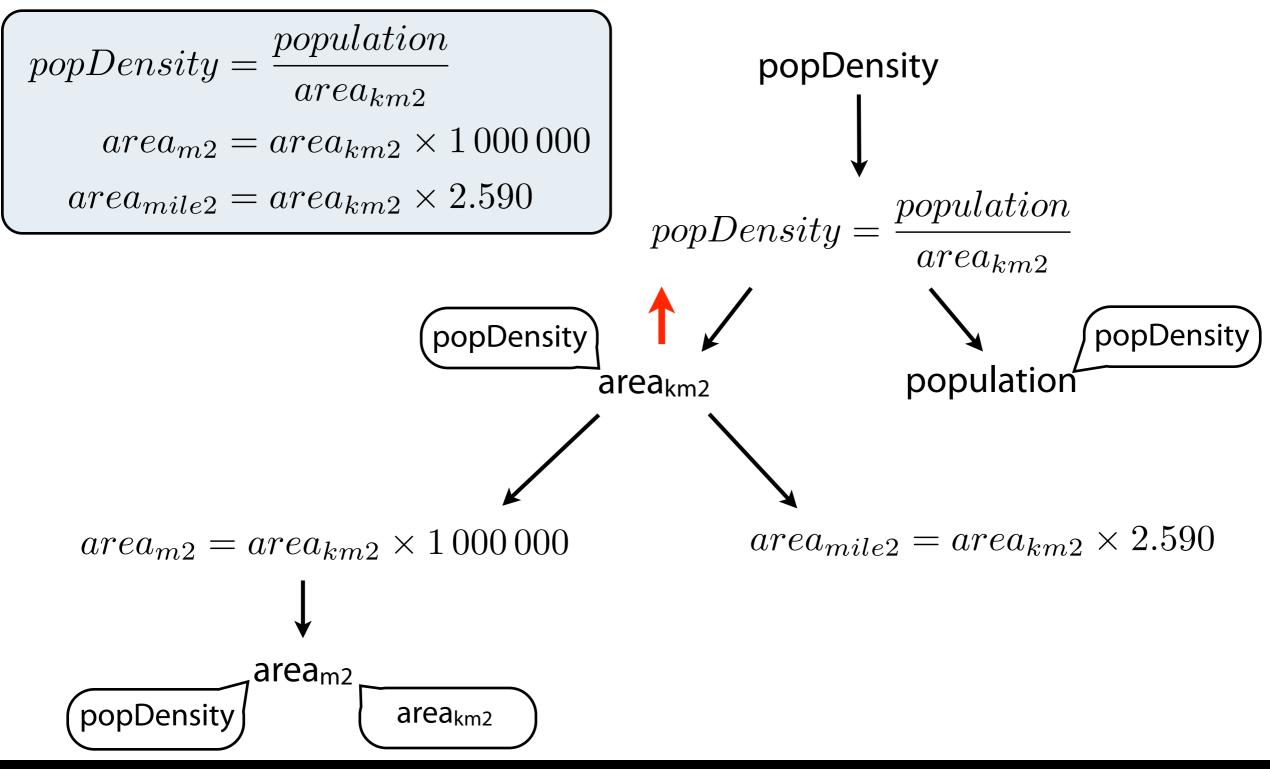


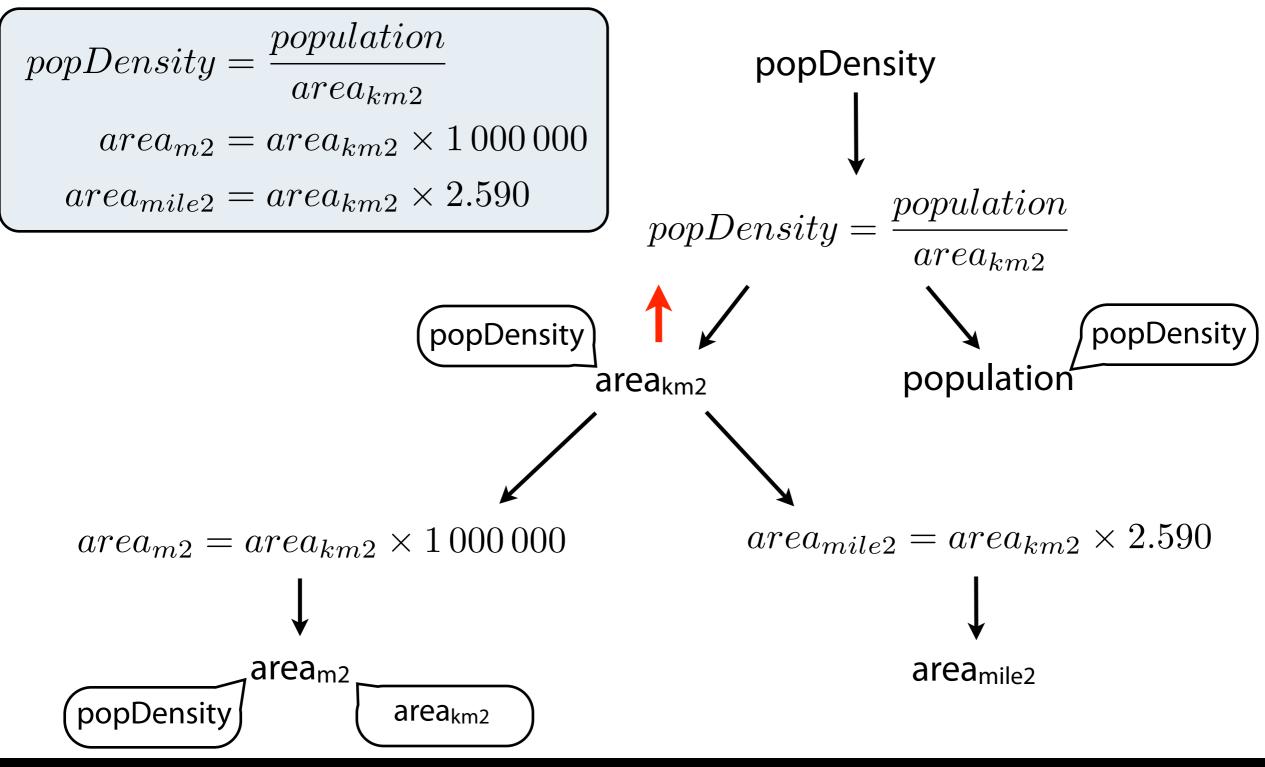


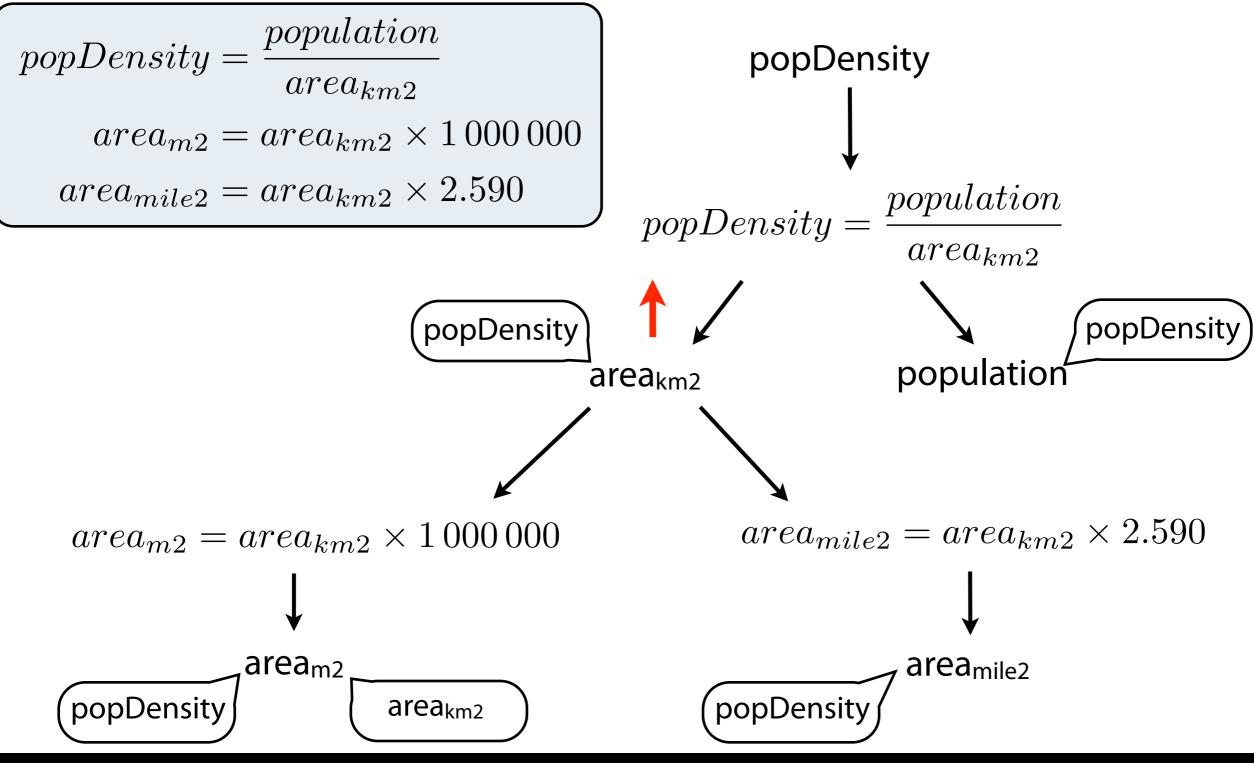


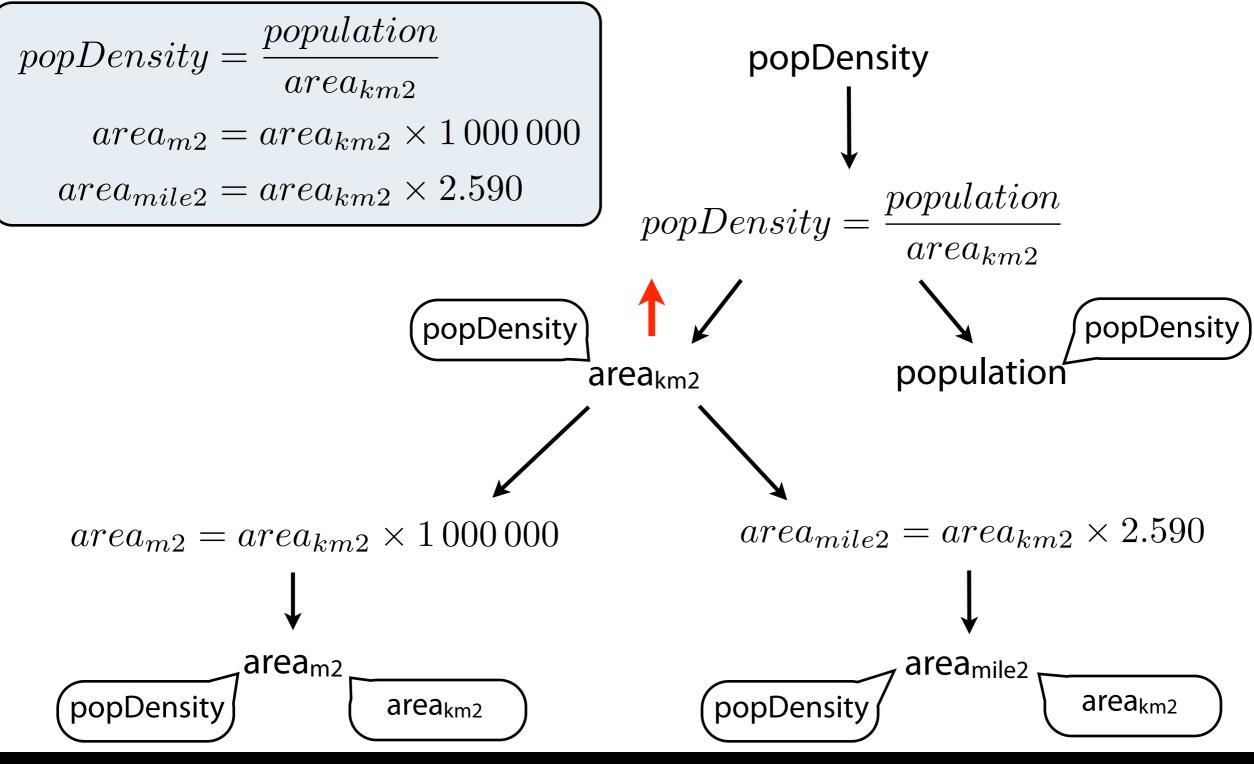




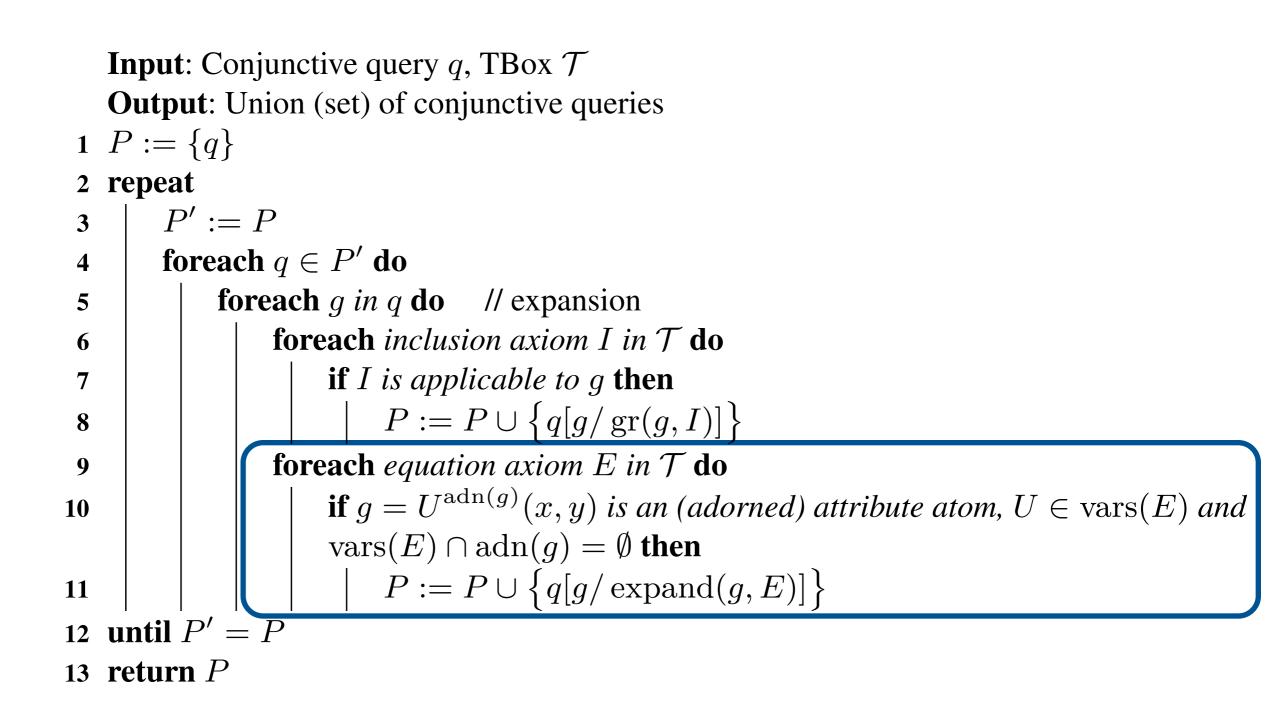








Extend the DL-Lite PerfectRef algorithm by equations and adorned attributes



PerfectRef with adorned attributes

query rewriting

Equations

$$E1: popDens = \frac{population}{areakm2}$$
$$E2: aream2 = areakm2 \times 1\,000\,000$$

Original query
 popDens(montpellier, X)

PerfectRef with adorned attributes

query rewriting

Equations

$$E1: popDens = \frac{population}{areakm2}$$
$$E2: aream2 = areakm2 \times 1\,000\,000$$

- Original query
 popDens(montpellier, X)
- Step 1: Expand popDens by E1 $population^{\{popDens\}}(montpellier, P),$ $areakm2^{\{popDens\}}(montpellier, A), X = P/A$

PerfectRef with adorned attributes

query rewriting

Equations

$$E1: popDens = \frac{population}{areakm2}$$
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- Original query
 popDens(montpellier, X)
- Step 1: Expand popDens by E1 $population^{\{popDens\}}(montpellier, P),$ $areakm2^{\{popDens\}}(montpellier, A), X = P/A$
- Step 2: Expand area by E2
 population^{popDens}(montpellier, P),
 aream2^{popDens,area}(montpellier, A1), A = A1 * 1000000, X = P/A

PerfectRef^E is sound but incomplete in general conditions for completeness

- ABox is data-coherent with the TBox model of each object has at most one value per attribute attribute inclusions must also be considered
- For data-coherent ABoxes wrt. the TBox and rewritten SPARQL queries (free of non-distinguished variables)
 PerfectRef^E is sound and complete

PerfectRef^E is sound but incomplete in general

conditions for completeness

:M dbp:area_km2 I . :M dbp:area_mi2 2.590 .

- ABox is data-coherent with the TBox model of each object has at most one value per attribute attribute inclusions must also be considered
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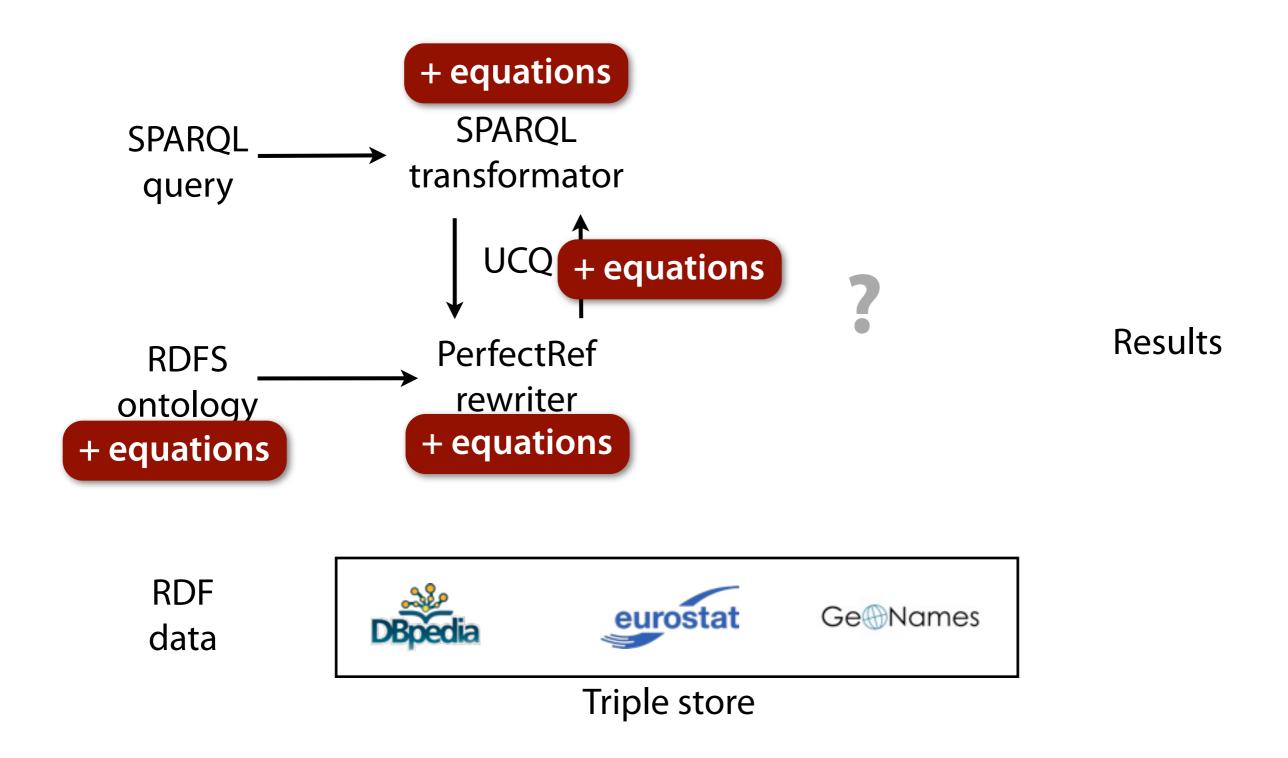
conditions for completeness

:M dbp:area_km2 I . :M dbp:area_mi2 2.590

:M dbp:area_km2 I . :M dbp:area mi2 2.6 .

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 PerfectRef^E is sound and complete

RDFS with Attribute Equations via SPARQL Rewriting the big picture



Rewrite SPARQL queries by PerfectRef

- SPARQL basic graph patterns (BGPs) the fundamental building block for graph pattern matching
- BGPs can be expressed by conjunctive queries [Perez et al., 2009] no variables as predicates :Montpellier ?prop "Montpellier" no variables for classes :Montpellier rdf:type ?class
- Convert BGPs to CQs, rewrite CQs to UCQs, convert UCQs to SPARQL
- SPARQL translator rewrite each BGP independently by PerfectRef^E
- Variable assignments are rewritten to SPARQL 1.1 BIND

Rewrite SPARQL query by PerfectRef rewritten SPARQL query

- CQ 1: popDens(montpellier, X)
- CQ 2: $population^{\{popDens\}}(montpellier, P),$ $area^{\{popDens\}}(montpellier, A), X = P/A$
- ▶ CQ 3: $population^{\{popDens\}}(montpellier, P), A = A1 * 1000000,$ $aream2^{\{popDens, area\}}(montpellier, A1), X = P/A$

SELECT ?X WHERE

{ :Montpellier dbo:populationDensity ?X . }

Rewrite SPARQL query by PerfectRef rewritten SPARQL query

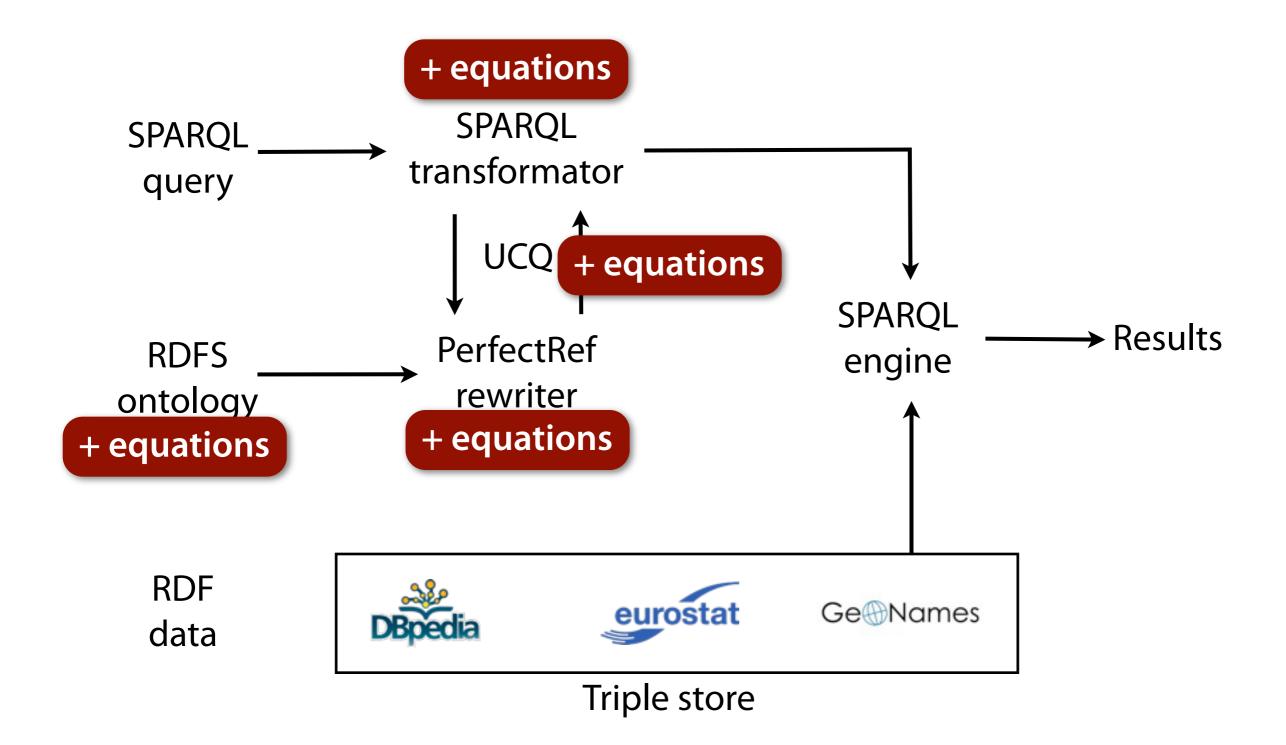
• CQ 1: popDens(montpellier, X)

}

- CQ 2: $population^{\{popDens\}}(montpellier, P),$ $area^{\{popDens\}}(montpellier, A), X = P/A$
- ► CQ 3: $population^{\{popDens\}}(montpellier, P), A = A1 * 1000000,$ $aream2^{\{popDens, area\}}(montpellier, A1), X = P/A$

```
SELECT ?X WHERE {
    { :Montpellier dbo:populationDensity ?X . }
    UNION
    { :Montpellier dbo:populationTotal ?p ; dbp:areaTotalKm ?a .
        BIND (?p/?a as ?X) }
    UNION
    { :Montpellier dbo:populationTotal ?p ; dbo:area ?a2 .
        BIND (?a2/1000000 as ?a) BIND (?p/?a as ?X) }
```

RDFS with Attribute Equations via SPARQL Rewriting the big picture



How does the rewriting algorithm perform on real world data?

 Collected data about cities from several sources (e.g., DBpedia, Eurostat) 254 081 triples for 3161 city contexts inconsistent and consistent dataset

- 6 equations, 2 subProperties and 1 subClass axioms
- 4 different queries

3 implementations

Jena forward rules with ARQ Jena forward rules with ARQ with noValue Rewriting with ARQ

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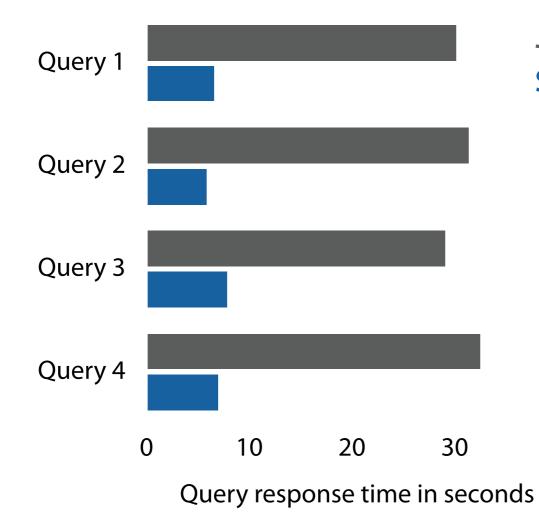
(?city :area ?ar) (?city :populationDensity ?pd) product(?ar, ?pd, ?p) noValue (?city, :populationDensity) -> (?city :population ?p)

Jena forward rules with ARQ Jena forward rules with ARQ with noValue **Rewriting with ARQ**

Jena rules with ARQ gives no results

- n rules for an equation in n variables
- Forward chaining implementation
- No query returned any result within 10 minutes
- Even for reduced dataset

Rewriting is significantly faster than Jena rules with noValue



Jena rules with noValue SPARQL query rewriting

Conclusions

- Reasoning about equations on numerical properties is important and feasible lots of numeric open data available
- Rule engines are not well suited for such attribute equations especially on real world data
- Query rewriting enables such reasoning on top of off-the-shelf SPARQL engines also possible on public SPARQL endpoints
- Query rewriting can be significantly faster than forward chaining rule engines